CMB: The ultimate test for theoretical models aiming at describing the very early universe

Mairi Sakellariadou

Section of Astrophysics, Astronomy, and Mechanics,
University of Athens, Panepistimiopolis, GR-15784 Zografos, Hellas.

February 1, 2008

Abstract

In this talk, I will illustrate how one can use the cosmic microwave background anisotropy measurements, in order to test theoretical models aiming at describing the early universe.

1 Introduction

The cosmic microwave background (CMB) anisotropy provides a powerful test for theoretical models aiming at describing the early universe. The characteristics of the CMB anisotropy multipole moments, and more precisely the position and amplitude of the acoustic peaks, as well as the statistical properties of the CMB anisotropy, can be used to discriminate among theoretical models, as well as to constraint the parameters space. CMB anisotropies are characterized by their angular power spectrum C_ℓ, which is the average value of the square of the coefficients of a spherical harmonic decomposition of the measured CMB pattern.

The position and amplitude of the acoustic peaks, as found by the CMB measurements — and in particular by the BOOMERanG [1], MAXIMA [2], and DASI [3] experiments — are in disagreement with the predictions of topological defects models. More precisely, defects models lead to the following predictions:

• Global O(4) textures predict the position of the first acoustic peak at $\ell \simeq 350$ with an amplitude ~ 1.5 times higher than the Sachs-Wolfe plateau [4].

• Global O(N) textures in the large N limit lead to a quite flat spectrum, with a slow decay after $\ell \sim 100$ [5]. Similar are the predictions of other global O(N) defects [6, 7].

• Local cosmic strings predictions are not very well established and range from an almost flat spectrum [8] to a single wide bump at $\ell \sim 500$ [9] with extremely rapidly decaying tail.

In conclusion, the latest CMB anisotropy measurements rule out pure topological defects models as the origin of initial density fluctuations.

As another example, I would like to mention the case of the pre-big-bang model (PBB), which is a particular cosmological model inspired by the duality symmetries of string theory. In an isotropic PBB model with extra dimensions, the amplification of Kalb-Ramnond axion vacuum fluctuations can lead to large-scale temperature anisotropies [10]. Within this model,

the perturbations induced by massless or very light Kalb-Ramond axions lead to a slightly blue spectrum of isocurvature perturbations [10], which in a closed universe with considerable cosmological constant, can fit the CMB data [11]. However, I believe that the existence of the isocurvature hump [11] can lead to an inconsistency between the predictions of the PBB model and the CMB measurements, at least within the current version of this cosmological model. It is not yet clear whether variations of the PBB model can cure this potential inconsistency between theoretical predictions and measurements [12].

The inflationary paradigm is at present the most appealing candidate for describing the early universe. However, inflation is not free of open questions and I would like to mention the following three types of issues, which represent a kind of open questions for any inflationary model.

• It is difficult to implement inflation in high energy physics. More precisely, the inflaton potential coupling constant must be very low in order to reproduce the CMB data. This is related to the question of deciding which kind of inflationary model is the more natural one.

• The quantum fluctuations are typically generated from sub-Planckian scales and therefore one should examine the validity of the theoretical predictions based upon standard quantum mechanics. Recent studies [13] seem to indicate that inflation is robust to some changes of the standard laws of physics beyond the Planck scale.

• It is almost always assumed that the initial state of the perturbations is the vacuum. The proof of such a hypothesis, if it exists, should rely on full quantum gravity, a theory which is still lacking. If the initial state is not the vacuum, this would imply a large energy density of inflaton field quanta, not of a cosmological term type [14]. Thus, non-vacuum initial states lead to a back-reaction problem, which has not been calculated yet.

In this presentation, I would like to briefly comment on the first and third types of issues, addressed in the context of inflation.

2 Choice of the inflationary model

Since inflation provides, at present, the most appealing candidate for describing the early univerese, we face the choice of the more physical inflationary scenario, within a rather large variety of different scenaria. Following the philosophy that the more natural cosmological model is the one which arises from particle physics theories, as for example superstring theories, we will see that topological defects can still play a role for the measured CMB anisotropies. In addition, within the context of the CMB anisotropy measurements, one should always keep in mind the problem of the degeneracy between various cosmological parameters. In what follows in this section, I will comment on a new degeneracy apparently arising in the CMB data, that would be due to a small — but still significant — contribution of topological defects.

In many particle physics based models, inflation ends with the formation of topological defects, and in particular cosmic strings [15]. Moreover, cosmic strings are predicted by many realistic particle physics models. Thus, even though the current CMB anisotropy measurements seem to rule out the class of generic topological defects models as the unique mechanism responsible for the CMB fluctuations, it is conceivable to consider a mixed perturbation model, in which the primordial fluctuations are induced by inflation with a non-negligible topological defects contribution.

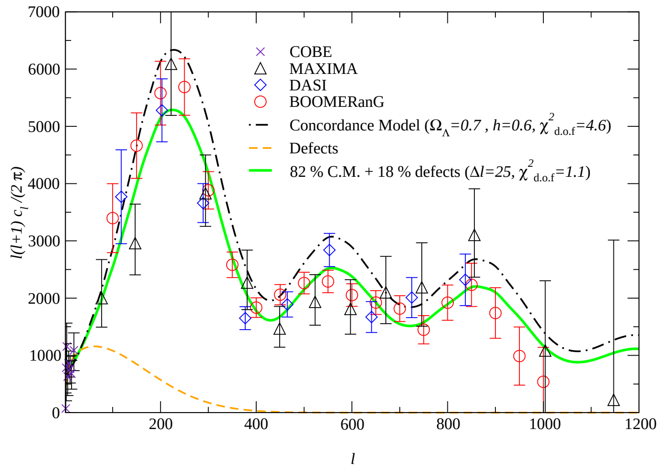

Figure 1: $\ell(\ell+1)C_\ell$ versus ℓ for three different models. The upper dot-dashed line represents the prediction of a ΛCDM model, with $n_{\rm S}=1$, $\Omega_\Lambda=0.7$, $\Omega_{\rm m}=0.3$, $\Omega_{\rm b}=0.05$ and $h=0.6$. The lower dashed line is a typical string spectrum. Both lines are normalized at the COBE scale (crosses). Combining both curves with the parameter α produces the solid curve, with a χ^2 per degree of freedom slightly above unity. The string contribution is about 18% of the total.

We consider [16] a model in which a network of cosmic strings evolved independently of any pre-existing fluctuation background, generated by a standard cold dark matter with a non-zero cosmological constant (ΛCDM) inflationary phase. As we shall restrict our attention to the angular spectrum, we can remain in the linear regime. Thus,

$$C_\ell = \alpha C_\ell^{\rm I} + (1-\alpha) C_\ell^{\rm S} \ , \tag{2.1}$$

where $C_\ell^{\rm I}$ and $C_\ell^{\rm S}$ denote the (COBE normalized) Legendre coefficients due to adiabatic inflation fluctuations and those stemming from the string network respectively. The coefficient α in Eq. (2.1) is a free parameter giving the relative amplitude for the two contributions. We have to compare the C_ℓ, given by Eq. (2.1), with data obtained from CMB anisotropy measurements. In this preliminary work, we do not vary $C_\ell^{\rm S}$ characteristics and simply use the model of Ref. [7].

At this point, we should mention that, strictly speaking, the anisotropy power spectrum reported in Ref. [7] concerns theories of global defects. However, we believe that the spectrum of Ref. [7] exemplifies the power spectra of both global and local cosmic strings. In a future work [17] we expect to have the C_ℓ induced by cosmic strings, employing a more acurate numerical simulation of a cosmic strings network. Figure 1 shows the two uncorrelated spectra (ΛCDM model, strings) as a function of ℓ, both normalized on the COBE data, together with the weighted sum. One clearly sees that neither the upper dot-dashed line — which represents the prediction of a ΛCDM model, with $n_{\rm S}=1$, $\Omega_\Lambda=0.7$, $\Omega_{\rm m}=0.3$, $\Omega_{\rm b}=0.05$ and $h=0.6$ — nor the lower dashed line — which is a typical string spectrum — fit the latest BOOMERanG, MAXIMA and DASI data (circles, triangles and diamonds respectively). The best fit, having $\alpha\sim 0.82$, yields a non-negligible string contribution, although the inflation produced perturbations represent the dominant part for this spectrum.

This mixed perturbation model we presented here, leads to the following two conclusions [16]:

• It seems still a bit premature to rule out any contribution of cosmic strings to the CMB anisotropy measurements, even though we are rather confident to conclude that pure topological

defects models are excluded as the mechanism of structure formation.
• There is some degree of degeneracy between the model with a string contribution and the one without strings but with more widely accepted cosmological parameters. We thus suggest to add the string contribution as a new parameter to the standard parameters space.

3 Non-vacuum initial states for cosmological perturbations of quantum mechanical origin

Here, I would like to briefly comment on the choice of the quantum initial state of the perturbations. In the literature, it is almost always assumed that the initial state of the perturbations is the vacuum. The proof of such a hypothesis, if it exists, should rely on full quantum gravity, a theory which is still lacking. The most convincing argument in favor of a vacuum initial state is the fact that non-vacuum initial states imply, in general, a large energy density of inflaton field quanta, not of a cosmological term type [14]. Thus, one should calculate the back-reaction effect on the background, but such a computation, even though it is in principle possible, it has never been performed. Here, instead of relying on purely theoretical arguments, we will try to determine [18], [19] whether the vacuum state is the only state compatible with the observations.

From the theoretical side, the simplest way to generalize the vacuum initial state, which contains no privileged scale, is to consider [18] an initial state with a built-in characteristic scale, $k_{\rm b}$. In a band localized around the prefered scale, the state contains n quanta, whereas it is still the vacuum elsewhere. More precisely, we define the state

$$|\Psi_1(\sigma,n)\rangle \equiv \bigotimes_{\mathbf{k}\in\mathcal{D}(\sigma)} |n_{\mathbf{k}}\rangle \bigotimes_{\mathbf{p}\notin\mathcal{D}(\sigma)} |0_{\mathbf{p}}\rangle, \tag{3.2}$$

where the domain $\mathcal{D}(\sigma)$, in momentum space, is such that, if $\mathbf{k}$ is bewteen 0 and σ, $\mathcal{D}$ is filled by n quantas, otherwise $\mathcal{D}$ is empty. Since the transition between the emplty and the filled modes is sharp, to "smooth out" the state $|\Psi_1\rangle$, we define a new state $|\Psi_2\rangle$ as a quantum superposition of $|\Psi_1\rangle$. In other words,

$$|\Psi_2(n)\rangle \equiv \int_0^{+\infty} \mathrm{d}\sigma g(\sigma)|\Psi_1(\sigma,n)\rangle, \tag{3.3}$$

where $g(\sigma)$ is a given function which defines the privileged scale $k_{\rm b}$. Assuming that the state $|\Psi_2\rangle$ is normalized, we have $\int_0^{+\infty} g^2(\sigma)\mathrm{d}\sigma = 1$.

As it was mentioned earlier, a choice of non-vacuum initial states suffers from the back-reaction problem [14]. Namely, we have the requirement that the background energy density must be larger than the energy density due to the quantum fluctuations in the state $|\Psi_2\rangle$. A back-to-the-envelope estimation [19] indicates that a model with approximately 60 e-foldings does not suffer from the back-reaction problem. Thus, there is still a (small) possibility that non-vacuum initial states are compatible with inflation.

A robust prediction of all models for which the initial state is not the vacuum is the non-Gaussian character of the induced perturbations. For models with a prefered scale, the three point (and any higher-order odd-point) correlation function vanishes, whereas the four-point (and any higher-order even-point) correlation function does not satisfy Gaussian statistics [18]. We will thus calculate [19] the fourth-order moment (kurtosis) for large angular scales, for which the source of non-Gaussianity can only be primordial. Performing a rather lengthy calculation,

for a given ansatz for the function $g(\sigma)$ which defines the previleged scale, we obtain [19] the normalised excess of kurtosis $\mathcal{Q}$,

$$\mathcal{Q} \equiv \left\langle \left[\frac{\delta T}{T}(\mathbf{e})\right]^4 \right\rangle \Big/ \left\langle \left[\frac{\delta T}{T}(\mathbf{e})\right]^2 \right\rangle^2 - 3 \; , \tag{3.4}$$

which we compare to the cosmic variance, calculated for a Gaussian field [20], namely

$$\mathcal{Q}_{\rm CV} \simeq \pm\sigma_{\rm CV} \Big/ \left\langle \left[\frac{\delta T}{T}(\mathbf{e})\right]^2 \right\rangle^2 \; , \tag{3.5}$$

where

$$\sigma^2_{\rm CV} = \left\langle \left[\frac{\delta T}{T}(\mathbf{e})\right]^8 \right\rangle - \left\langle \left[\frac{\delta T}{T}(\mathbf{e})\right]^4 \right\rangle^2 \; . \tag{3.6}$$

The comparison of the excess of kurtosis, calculated from our model, to the cosmic variance, will indicate the feasibility of detecting this non-Gaussian signal. Our analysis [19] shows that the excess of kurtosis is smaller than the cosmic variance. Thus, since we obtained a signal-to-(teoretical) noise smaller than 1, we conclude that the non-Gaussian signal, predicted by the choice of non-vacuum initial states for cosmological perturbations, is not detectable.

The implication of this analysis is quite important for inflation. We have examined a possible model, in the context of inflation, which leads to non-Gaussian statistics of the CMB anisotropy. We showed that this non-Gaussian signature is much smaller than the cosmic variance, thus it remains undetactable. We therefore conclude [19] that the prediction of Gaussian statistics within the context of inflation is quite robust.

4 Conclusions

In this talk, I presented how using the measurements of the CMB anisotropies, one examines the validity of cosmological models aiming at describing the early universe. I explained why topological defects cannot be accepted as the source of primordial fluctuations which gave rise to structure formation. I also mentioned why the present version of the pre-big-bang model may soon by in contradiction with the CMB data. I then showed that a cosmological model where perturbations are induced by inflation with a non-negligible topological defects contribution, is compatible with the latest CMB anisotropy measurements. In addition, as I said, such a model can lead to a new degeneracy in the data. Finally, I discussed why the prediction of inflation, namely that the CMB anisotropies obey Gaussian statistics, is indeed robust.

Acknowledgments

It is a pleasure to thank the organizers of the Cosmo 01 meeting for inviting me to present this work. I would also like to thank all my colleagues, with whom I collaborated on the projects I presented here.

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